CONCEPT OF THE QUANTITATIVE RISK GENERALIZED MODEL ASSESSMENT

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Abstract

The paper presents an analysis of current trends in methods of modeling uncertainty, with a particular emphasis on the theory of monotone measures and the fuzzy sets theory. Subsequently, a critical review of the concept of defining and evaluating the concept of risk was conducted. On this basis, the definitions of risk used so far have been classified, and a new, generalized idea of a qualitative and quantitative perspective of the concept of undertaking implementation risk has been proposed. The designed model allows for the assessment of risk also in unusual situations, for which there are no historical data that make it possible to use classical methods in the area of he set theory, mathematical statistics and the probability calculus, which forces one to use subjective expert assessments.

Key words: risk, uncertainty, fuzzy sets, theory of possibility, theory of measures.

Introduction

The issue of assessing the risk of implementing economic undertakings of manufacturing and service nature, in particular global ones, has been becoming more and more important in recent years. It results from the efforts to minimize costs by continuous processes of Lean Manufacturing, Lean Logistics, and Lean Management and the increasing number and intensity of external threats. Reducing the number of suppliers according to the "4S" principle (a Single Source Supply Strategy), introducing Just In Time Manufacturing system to a greater extent, minimizing the level of minimal buffers, and configuring tightly connected supply chains have resulted in a significant increase in the level of risk of process continuity. This risk is greater when the information about the possible risks that may interfere with the supply process and their possible impact is burdened with greater uncertainty.

In the case of predictable, recurring threats, it is possible to "tame" uncertainty, and at the same time limit risk, by means of statistical methods, based on the probability calculus, the theory of stochastic processes, and mathematical statistics. By contrast, so far there have been no effective methods and tools useful to assess the risk associated with a new type of threat of unique character, related to the forces of nature, or intentional, hostile interference of the human factor. Untypical risks are especially dangerous, and their impact can be adverse for all people participating in the implementation of the undertaking.

These risks are sometimes called "Black Swan" events and have the following characteristics (Taleb 2010):

- uniqueness, going beyond expectations based on previous experiences and a priori unpredictability,
- extremely large impact of the event,
- a possibility to explain the reasons and the method of forecasting after its occurrence (a posteriori).

The effect of "Black Swan" is also described in the literature ad LSLIRE (Large Scale, Large Impact Rare Event).

The purpose of this paper is to propose a universal concept of quantitative assessment of risk related to the occurrence of such events, and resulting in disrupting process continuity on a large scale, which is particularly important for example in global chains. First, the methods for modeling uncertainty in terms of their applicability to assess this type of risk will be reviewed, and then the concept of a universal model will be presented, enabling a quantitative risk assessment, also in unusual situations that have not yet appeared in the past.

1. Methods for uncertainty modeling

1.1. Theories of uncertainty

The starting point for considering uncertainty may be GIT – the Generalized Information Theory, proposed by Klir (2004). Compared to the classical information theory created by Shannon (1948), based on concepts of probability and entropy, it has a much more universal character. GIT is the result of two significant mathematical generalizations:

- the classical theory of additive measures to the theory of monotone measures,
- the classical theory of crisp sets to a more general theory of fuzzy sets.

The first generalization, which started in the early 1950s, extends additive measures to less restrictive monotonic measures, characterized by more diverse features. The second one, introduced in the 1960s, expands the language of the classical set theory into a more universal language of fuzzy sets, allowing the use of vague linguistic terms. The theory of uncertainty of a given type is formed by choosing the appropriate language (e.g. based on the theory of fuzzy sets) and expressing uncertainty by means of specific monotone measures (e.g. based on the theory of probability).

The classical information theory is based on the theory of probability or alternatively, the theory of possibility, applied to classical sets. And applying the probability function or possibility function to standard fuzzy sets enables the creation of new, more general theories of information. Similarly, remaining at classical sets, we can apply various non-additive monotone measures (e.g. Sugeno's λ measure, Dempster-Shafer measure, Choquet integral of n-th order or the general lower and upper probability function), creating new theories.

There are, therefore, a lot of formal theories of uncertainty. Each of them is more or less general, and any two theories at the same level of generality may not be mutually comparable. Each particular problem requires the use of such a theory, which would make it possible for a decision-maker to express his or ignorance and to protect against ignoring any information, relevant in a given situation. Currently, within the framework of the so-called imperfect knowledge trend, efforts are made to create GTU – the Generalized Theory of Uncertainty, which would go beyond the classical theory of probability and classical set theory, would characterize each type of uncertainty and work at four levels: formalization , computational tools, measurement and methodology (Zadeh 2005).

In each of the above theories, uncertainty is represented by the so-called uncertainty function, assigning each possible realization from the set a number from the interval [0,1], which determines the degree of certainty that a specific opportunity arises. Examples of uncertainty functions include: the probability function, the possibility function, the function of faith and credibility or function of the lower and upper probability. In each theory, the uncertainty function meets certain requirements that differentiate the various theories. The measure of uncertainty for a specific type of theory is the functional that assigns a non-negative real number to each function. Typical examples of uncertainty measures are the Shannon entropy (Shannon 1948) and Hartley measure (Hartley 1928). A functional representing the uncertainty measure must meet a number of requirements. Admittedly, mathematical formalization of each of the requirements depends on the theory used, however, these requirements can be represented in the general form as:

- additivity – the uncertainty in the total data representations is equal to the sum of uncertainty of individual representations of data,
- subadditivity – the uncertainty in the total representation of data cannot be greater than the total uncertainty of the sum of individual data representation,
- range – the uncertainty is contained in the interval (0, M), where 0 is related to the function that describes the complete certainty and M depends on the size of the set used and the selected unit of measure,
- continuity – the functional must be continuous,
- expansibility – developing a set of alternatives by adding alternatives cannot change the level of uncertainty
- consistency – if uncertainty can be calculated in different ways (allowed in the method), the result must be the same,
- monotonocity – if the data form an increasing series, their measure of uncertainty also increases and vice versa, if the data in a series are decreasing, uncertainty also decreases,
- coordinate invariance – a measure of uncertainty cannot change with the isometric coordinate transformations.

These requirements must be fulfilled by all types of uncertainty that exist in the theory.

Uncertainty of information can be expressed through measures of probability, possibility and necessity, faith and credibility. Measures of possibility, necessity, faith (conceivability) and credibility are dual, i.e., if the event is necessary, the counter-event is impossible, if the event is credible, the counter- event is inconceivable.

There are three main principles of uncertainty management:

- the principle of minimum uncertainty – we accept only those solutions for which the loss of information (resulting from simplifications, transformations conflict solutions) is minimal, i.e. we choose solutions with a minimum of uncertainty,
- the principle of maximum uncertainty – we accept all solutions, after making sure that the information that raises doubt is reliable,
- the principle of uncertainty invariance – the level of uncertainty should be kept at each transition from one mathematical approach to another.

1.2. Formalized languages

Currently, three formalized languages are used to describe sets: CST – the Classical Sets Theory, SFST – the Standard Fuzzy Sets Theory and

NFST – the Nonstandard Fuzzy Sets Theory. The first two theories are thoroughly described in the literature, are well-developed and widely used, while the last one is a relatively new theory and not yet fully developed.

1.2.1. The classical set theory

In the classical set theory, it is assumed that each element of the considered space X belongs to either set A ($x \in A$) defined on the space X ($A \in P(X)$), or it complements the set A ($x \in \bar{A}$), that is, no element can belong simultaneously to both sets. The characteristic function (membership function) of the set A is

$$m_A : X \to \{0,1\} \text{ and } m_A = \begin{cases} 1 \text{ for } x \in A \\ 0 \text{ for } x \notin A \end{cases} \quad \text{for each } x \in X \tag{1.1}$$

Two sets are equal only when every element of one of them is the element of the other and vice versa. Two sets of the same number of elements are called equinumerous.

The interference based on binary logic and classic sets is simple and unambiguous, but in many cases insufficient to describe the complex reality.

1.2.2. The standard fuzzy sets theory

The concept of fuzzy sets was introduced by Zadeh (1965) as a generalization of the classical set theory. In the case of fuzzy sets, each element of space X can belong partially to a set A, and partly to its complement $\bar{A}$. Fuzzy sets are defined by the membership function corresponding to the function characteristics of classical sets. Each element of the set X has the assigned value that defines the degree of membership to the fuzzy set. Membership of standard fuzzy sets is in the range [0,1] and if the maximum value equals 1, we deal with normal fuzzy sets. Thus, the membership function of the set X is:

$$\mu_A : X \to [0,1] \tag{1.2}$$

We can distinguish three cases here:

a) $\mu_A(x) = 1$ – means full membership in the fuzzy set A
b) $\mu_A(x) = 0$ – means the lack of membership in the fuzzy set A
c) $0 < \mu_A(x) < 1$ – means a partial membership in the fuzzy set A

A fuzzy set A is contained in the fuzzy set B only when $\mu_A(x) < \mu_B(x)$ for each $x \in X$, and the fuzzy set A equals the fuzzy set B only when $\mu_A(x) = \mu_B(x)$. The complement of the set A is a fuzzy set $\bar{A}$ with a membership function $\mu_{\bar{A}} = 1 - \mu_A$.

Although the inference based on the fuzzy set theory and multi-valued logic is more complex and less intuitive, however thanks to widely available computer tools supporting the process of the fuzzy inference it is becoming more common (Klir, Yuan 1995).

1.2.3. Non-standard fuzzy sets theories

A further generalization of the sets theory are non-standard fuzzy sets. These include, inter alia, fuzzy sets of type 2, for which the degrees of membership are also fuzzy sets (Mendel 2001). In this case, the membership function is expressed by:

$$\mu_A : X \to [0,1]^{[0,1]} \tag{1.3}$$

A simpler variant of the above sets are interval-valued fuzzy sets, in which for every x a membership function μ_A has values in the interval [0, 1].

1.3. Monotonic measures

There is a set X and a non-empty family of sets C which are subsets of X and containing the empty set $\varnothing$ and the set X. A regular monotonic measure g on $\langle X, C \rangle$ is function:

$$g : C \to [0,1] \tag{1.4}$$

satisfying the following conditions:

1. $g(\varnothing) = 0$ and $g(X) = 1$ (boundary conditions)
2. for each $A, B \in C$, if $A \subseteq B$, then $g(A) \leq g(B)$ (monotonicity)
3. for each $A_1 \subseteq A_2 \subseteq \ldots$ increasing sequence there is:

$$\text{if } \bigcup_{i=1}^{\infty} A_i \in C\text{, then } \lim_{i \to \infty} g(A_i) = g(\bigcup_{i=1}^{\infty} A_i)$$

   continuity from below

4. for each decreasing sequence $A_1 \supseteq A_2 \supseteq \ldots$ there is:

$$\text{if } \bigcap_{i=1}^{\infty} A_i \in C\text{, then } \lim_{i \to \infty} g(A_i) = g(\bigcap_{i=1}^{\infty} A_i)$$

   continuity from above

If we assume that $A, B \in C$ and $A \cap B = \varnothing$, in the theory of monotone measures, the following situations are possible:

1. $g(A \cap B) \geq g(A) + g(B)$ – superadditivity, or interaction (reinforcement) of A and B in relation to the assessed volume,

2. $g(A \cup B) = g(A) + g(B)$ – additivity, i.e. no dependence on A and B in relation to the assessed volume,
3. $g(A \cup B) \leq g(A) + g(B)$ – subadditivity, or incompatibility (weakening) of A and B in relation to the assessed volume.

1.3.1. Additive measures - the “numeric” probability

A classic tool in studying uncertainty is the probability theory. Applying it leads to assumptions, such as that the rate of uncertainty should be measurable (using simple methods) and have a numerical rating. Another assumption used in the studies of uncertainty is the independence of the system components. Uncertainty and uncertain information are modelled in this case also by a probability distribution. For a finite set *X* of mutually exclusive alternatives, a probability distribution function for each $x \in X$ is

$$p(x) \in [0,1] \qquad \sum_{x \in X} p(x) = 1 \tag{1.5}$$

1.3.2. Non-additive measures

A characteristic of non-additive measures is that the probability of the sum of mutually exclusive (independent) events does need not to equal probability of these events. This reflects a subjective assessment of probability by a decision-maker, which is not necessarily an objective probability. This condition is fulfilled by measures based on the possibility theory and the imprecise theory of probabilities.

The possibility theory

The concept of possibility and necessity is the oldest and most fundamental concept of measures, derived from the ideas of Aristotle. However, the theoretical basis for this concept was developed by Zadeh in the 1970s (Zadeh 1978). It is assumed that there is a set of mutually exclusive alternatives *X*. The basic information that we can obtain on the set *X* (based on different types of tests) is the information that certain alternatives from a set *X* are impossible. After rejecting impossible alternatives, we obtain a set of possible alternatives *E*, which is a subset of *X*. The characteristic function of the set *E* (also known as the basic function of possibility) is:

$$r_E(x) = \begin{cases} 1 & \text{for } x \in E \\ 0 & \text{for } x \notin E \end{cases} \tag{1.6}$$

A function of possibility defined on a power set *P* (*X*) is given by the formula:

$$Pos_E(A) = \max_{x \in A} r_E(x) \text{ for each } A \in P(X) \tag{1.7}$$

A real alternative may belong to a set *A* if *A* contains at least one element of the set *E*. Functions of necessity can be formulated as follows:

$$Nec_E(A) = 1 - Pos_E(\overline{A}) \quad \text{for each } A \in P(X) \tag{1.8}$$

The real alternative is necessary in A only when it is not possible that it is in a completion of *A*. An uncertainty measure of a finite set of possible alternatives *E* is Hartley's measure expressed by the formula:

$$H(Pos_E) = \log_2 |E| \tag{1.9}$$

The theory of imprecise probabilities

The theory of imprecise probabilities is used for experimental data burdened with uncertainty, for which it is difficult or even impossible to calculate the probability characteristics. A characteristic of all imprecise probabilities inaccuracy is that the data can be described by means of the low or high probability functions ($\underline{g}$ i $\overline{g}$). Functions $\underline{g}$ i $\overline{g}$ are regular monotonic measures and they fulfill the following conditions (Utkin 2004; Walley 1991):

$$\sum_{x \in X} \underline{g}(\{x\}) \le 1, \sum_{x \in X} \overline{g}(\{x\}) \ge 1$$

The upper probability is a subadditive measure, and the lower probability is a superadditive measure. The classic probability measure is a special case of imprecise probabilities, for which lower and upper probabilities are equal.

2. Modeling the risk of the undertakings implementation

The concept of risk is very broad and covers many areas of human activity, and therefore, it has a social character (Arnoldi 2009). In the most general terms, it is understood as a situation involving exposure to danger (Oxford English Dictionary 2014). Polish civil law defines risk as "the risk of loss burdening a victim". In applied sciences, it was adopted at the beginning of the 20th century that the risk is objectively dependent on subjective uncertainty, and a little later Knight (1924) published his uncertainty theory, dividing uncertainty into measurable one (i.e. risk), and non-measurable (uncertainty sensu stricto). Based on these assumptions, in the following years, a number of definitions of the concept of risk and their modifications appeared, which were successfully used to model risk when making decisions in specific areas of activity (e.g. Bac 2009; Bukowski, Stachowicz 2006; Bukowski, Karkula 2010; Jajuga 2009; Karkoszka 2013; Manikowski 2013; Szkutnik 2010).

An interesting summary of the search for a universal definition of risk is the work of Aven (2012), where there is a critical review of the literature on risk and based on it, the classification of the definition of risk was proposed, dividing it into 9 groups. Some of these definitions are now only of historical interest, so they will be omitted and only those will be discussed that are currently, according to the author, of fundamental importance. They have been divided into three classes and characterized below.

1. ***Risk as the expected value of the possible outcome (mostly losses).*** Historically it is the oldest quantitative approach to risk, attributed to the mathematician de Moivre (Damodarian 2009; de Moivre 1738) and it is still applied in the financial analysis and decision--making, when so-called decision trees are used. In this approach, a measure of risk is the product of probability of the event and the financial consequences of that event. This group can include the concept of Adams (1995), who defines risk as the product of the probability and utility of an event which may occur in the future.
2. ***Risk as a combination of the effects of the certain scenario implementation and probability of that scenario.*** This group includes a lot of concepts of risk assessment, but the proposal of Kaplan and Garrick (1981) is the most universal: they define risk as an ordered triplet (s_i, p_i, c_i), where si is a scenario, p_i is the probability of this scenario implementation, and c_i represents consequences (usually financial) of this scenario implementation, for i = 1, 2, 3, … n.
   Also, this group may include the definition recommended by the standard PN-N-18 002, which says that risk is a combination of frequency or probability of occurrence of an event causing risk and the consequences of this event.
3. ***Risk as a result of uncertainty of achieving the set objectives.*** This is the most general approach recommended by ISO 31000 version of the 2009. It is based on the trend proposed by Hardy already in 1923 (Hardy 1923), who understood the concept of risk as uncertainty about costs, losses or damages that may occur in the future. This approach allows for defining more clearly the “operational” concept of risk, depending on the specific character of the problem and perspective of its examination.

Based on the above discussions and taking into account the current possibilities of modelling uncertainty presented in point 2 of this paper, the following universal definition of risk is proposed, from both the quantitative and qualitative perspective:

A qualitative definition. Risk is defined as the possibility of obtaining the result (effect) of the undertaking implementation, different from the expected result.

A quantitative definition. A measure of risk R of the undertaking implementation X is an ordered triplet, consisting of:

- A set of graphs $\{G_i\}$ (e.g. networks or trees of events), representing the possible timing of the undertaking implementation X,
- A set of outcomes of the individual courses of actions $\{O_i\}$ expressed in the language of the classical set theory or fuzzy set theory,
- A set of monotonic measures (additive or non-additive) $\{P_i\}$ representing uncertainty (probability, chance, possibility) of the implementation of individual courses of the undertaking implementation process X.

This model can be written in the following form:

$$(X) = < \{G_i\}, \{O_i\}, \{P_i\} > \qquad \text{for } i = 1, 2, 3, \ldots \qquad (2.0)$$

In this approach, all three classes of the definition of risk – a), b) and c) – can be described by means of one universal model (3.1). And so in the case of the traditional approach (class a), the set of result $\{O_i\}$ for each implementation will be described in the language of the classical set theory, a set of uncertainty measures $\{P_i\}$ will be estimated based on the probability theory and mathematical statistics. However, in the most general case, for which possible courses of process (scenarios) are hypothetical, prediction of outcomes of these courses will be imprecise, and probability of their implementation impossible to estimate. Then:

- a set of possible scenarios can be described by means of a set of oriented fuzzy graphs (e.g. Pins et al. 1991),
- a set of outcomes will be described in the language of fuzzy sets (e.g. Bukowski, Felix 2005),
- a set of uncertainty measures corresponding to the outcomes, defined with the use of the possibility theory (e.g. a function of possibility and necessity).

Thus, adopting the model of risk in the form (3.1) will also allow a quantitative risk assessment for threats occurring very rarely, and in particular the type LSLIRE events (Large Scale, Large Impact Rare Event).

Conclusions

Economic undertakings, including the implementation of logistics processes, take place in the conditions of risk, and their management is based on available information, which in practice is usually incomplete and uncertain (Bukowski, Majewska 2004). Risk minimization requires that decision-makers properly assess the degree of uncertainty of the information. When there are numerous sets of historical data, the probabilistic methods, based on the probability calculus and mathematical statistics are used for these purposes.

However, in many cases, the decision maker has only few data, burdened with uncertainty, inaccuracy and incompleteness, which cannot be described in terms of probability. In these situations, for the quantitative risk assessment, it is advisable to apply the fuzzy sets theory and non-additive measures of uncertainty. This possibility is offered by a universal model of quantitative risk assessment, which has been proposed in this paper.

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